Department
of
Mathematics and Statistics

School of Mathematical and Physical Science

Syllabus of
Ph.D. (Mathematics) Entrance Test

Session 2023-24 onward

Date of BoS -12/12/2023

Doctor Harisingh Gour Vishwavidyalaya
(A Central University)
Sagar-Madhya Pradesh-470003

Passed by School Board Dated 13/12/23

Passed by Board of Studies Dated 12/12/23

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12/12/23
(Chairman, School Board
BOS

Ph.D. (Mathematics) Entrance Syllabus from Session 2019 onwards
Dr. Harisingh Gour Central University, Sagar (M.P)

Part A: Research Methodology

1. **Foundations of Research**
   Meaning, Objectives, Motivation, Utility. Concept of theory, empiricism, deductive and inductive theory. Characteristics of scientific method – Understanding the language of research – Concept, Construct, Definition, Variable. Research Process
2. **Problem Identification & Formulation**
   Research Question – Investigation Question – Measurement Issues – Hypothesis – Qualities of a good Hypothesis –Null Hypothesis & Alternative Hypothesis. Hypothesis Testing
3. **Research Design**
   Concept and Importance in Research – Features of a good research design. Exploratory Research Design – concept, types and uses. Descriptive Research Designs – concept, types and uses. Experimental Design: Concept of Independent & Dependent variables.
4. **Qualitative and Quantitative Research**
   Qualitative research – Quantitative research – Concept of measurement, causality, generalization, replication. Merging the two approaches.
5. **Awareness with MS-Word**, file operations, text editing operations, various table designs, insertion of tables in text, use of mathematical symbols and equation editor, Use page layout and Working practice with MS-Word. Awareness with MS Excel, worksheet creation, Computations using worksheet, Use of mathematical and statistical functions, Awareness about idenformation technology in research, working practice with MS-Excel

Part B: Mathematics

Analysis:

Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation. Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Linear Algebra:

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

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Algebra: Permutations, combinations, pigeon hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's ϕ-function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory.

Topology: basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

Ordinary Differential Equations (ODEs): Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs): Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Numerical Analysis: Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Calculus of Variations: Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Linear Integral Equations: Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Descriptive statistics, exploratory data analysis Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions.

Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case). Markov chains with finite and countable state space, classification of states.

Linear programming problem, simplex methods, duality. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.

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Member of School Board of Mathematical and Physical Sciences
On 13-12-2023
(For Department of Mathematics & Statistics)

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Prof. A.K. Saxena
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Deptt. of Mathematics
Maharaja Chhatrasal Univ.,
Chhatarpur- MP

Online consented
Prof. Narendra Pandey
External Member
Deptt. of Physics,
University of Lucknow,
Lucknow - UP

Online Consented
Prof. Kavishanker Varshney
External Member
Deptt. of Physics
D.S College, Aligrah

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Prof. Diwakar Shukla
Deptt. of Mathematics & Statistics

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Prof. R.K. Gangele,
Deptt. of Mathematics & Statistics

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Deptt. of Physics

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Deptt. of Pharmaceutical Science

Prof. R.K. Rawat
Deptt. of Math. & Statistics

13.12.23
Dr. Mahesh Kumar Yadav
Deptt. of Math. & Statistics

13/12/23
Prof. Ashish Verma
Head, Deptt. of Physics

13/12/23
Prof. Ashish Verma
Head, Deptt. of Computer Sc. & Appl.

on leave
Dr. Maheswar Panda
Deptt. of Physics

Mr. Kamal kant Ahirwar
Deptt. of Computer Sc. & Applications

13/12/23
Prof. Ashish Verma
Dean & Chairman, School Board of SMPS

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