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Power Spectral Density of FSK Modulation In Complex Binary System

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ABSTRACT

Power spectral density of frequency shift keying for a complex base band signal is being studied using complex binary representation. Frequency shift keying for complex signal has become simple and straight due to complex binary representation. The spectrum of FSK modulation is found suitable for digital mobile radio telephony.

Keywords: Complex Binary Representation, Power Spectral Density (PSD).

Introduction

Now a days digital transmission has become a challenging task among the scientists and engineers because of the use of computers for signal processing. One always requires modulator at transmitter and a demodulator to recover the base band signal in processing the signal from one computer or terminal to the other computer or terminal. There are three basic modulation techniques for the transmission of data. They are known as amplitude shift keying (ASK), frequency shift keying (FSK) and phase shift keying (PSK). We are interested only in FSK. Frequency shift keying has been studied extensively by different authors (Garber and Pursley, 1989, Choi, et al., 2002, Zhang, 2002, Natarajan, Nassar and Shattil, 2004, Leus et al., 2004). From spectral point of view FSK has been modified into minimum shift keying (MSK) (Pasupathy, 1979, Groner and Mcbride, 1976) sinusoidal frequency shift keying (SFSK) (Amoroso, 1976) and Gaussian minimum shift keying (GMSK) (Murota and Hirade, 1981).

In this paper our aim is to study the power spectral density of complex base band FSK modulation and demodulation using complex binary data. A brief review of complex binary representation alongwith the power spectral density is discussed. Finally, the conclusion of the paper is discussed.

Complex Binary Representation

Following (Pekmestzi, 1989) the complex number can be represented in complex binary digit as follows :

Let us consider the complex number $X = X_r + jX_i$ and the two's complement representations of X_r and X_i

$$Xr = -x_{ro} + \sum_{k=1}^{N-1} x_{rk} 2^{-k}$$

$$X_i = -x_{io} + \sum_{k=0}^{N-1} x_{ik} 2^{-k}$$

By using these equations, X becomes

$$X = -x_0 + \sum_{k=1}^{N-1} x_k 2^{-k} \qquad \text{...... (a)}$$

Where $x_k = x_{rk} + jx_{ik}$ ...... (b)

and $k = 0, 1, ----, N-1$

The variable x_k, called the complex binary digit, can take the values given in table-1.

Each complex bit x_k takes four possible values. Consequently, it must be represented by two bits, as shown in Table-1. So, the complex numbers are represented in a form similar to that of the real numbers in two's complement representation, as it can be seen in equation (a). This representation allows the development of algorithms for operations with complex numbers and the ability to describe these algorithms in the bit-level. It is natural that these algorithms and the corresponding circuits have great similarities to those for real numbers in two's complement form.

Power Spectral Density

Complex signal vectors representing the bits 1+j, 1, j and 0 respectively are given as

$$S_1(t) = A \exp j(\omega_c + \Delta\omega_1 + \Delta\omega_2)t$$
$$= A \exp j(\omega_1 t) \qquad \text{... ... (1)}$$

for $0 \le t \le T$

where $\omega_1 = \omega_c + (\Delta\omega_1 + \Delta\omega_2)$

and T is complex binary period

$$S_2(t) = A \exp j(\omega_2 t) \qquad \text{... ... (2)}$$

where $\omega_2 = \omega_c + (\Delta\omega_1 - \Delta\omega_2)$

$$S_3(t) = A \exp j(\omega_3 t) \qquad \text{... ... (3)}$$

where $\omega_3 = \omega_c - (\Delta\omega_1 - \Delta\omega_2)$

$$S_4(t) = A \exp j(\omega_4 t) \qquad \text{... ... (4)}$$

where $\omega_4 = \omega_c - (\Delta\omega_1 + \Delta\omega_2)$

In general

$$S_i(t) = A \exp j(\omega_i t) \qquad \text{... ... (5)}$$

where $i = 1, 2, 3$ and 4

Hence four symbol shaping functions are given as

$$\left.\begin{aligned} g_1(t) &= A \exp(\Delta\omega_1 + \Delta\omega_2)t \\ g_2(t) &= A \exp(\Delta\omega_1 - \Delta\omega_2)t \\ g_3(t) &= A \exp[-(\Delta\omega_1 - \Delta\omega_2)t] \\ g_4(t) &= A \exp[-(\Delta\omega_1 + \Delta\omega_2)t] \end{aligned}\right\} \qquad \text{... ... (6)}$$

Here, $g_1{}^*(t) = g_4(t)$

and $g_2{}^*(t) = g_3(t)$

So, $G_1(f) = A\int_{-T_b/2}^{+T_b/2} \exp j(\Delta\omega_1 + \Delta\omega_2)t.\exp(-j\omega t)dt$

Solving the integral, we have

$$G_1(f) = AT_b\left[\frac{\sin((\omega - (\Delta\omega_1 + \Delta\omega_2))T_b/2)}{(\omega - (\Delta\omega_1 + \Delta\omega_2))T_b/2}\right] \qquad \text{... ... (7)}$$

Energy spectral density is given by

$$\Psi_{g_1}(f) = |G_1(f)|^2 = A^2 T_b^2\left[\frac{\sin((\omega - (\Delta\omega_1 + \Delta\omega_2))T_b/2)}{(\omega - (\Delta\omega_1 + \Delta\omega_2))T_b/2}\right]^2 \qquad \text{... ... (8)}$$

Hence, power spectral density is written as

$$P_1(f) = \frac{\Psi_{g_1}(f)}{T_b} = A^2 T_b\left[\frac{\sin((\omega - (\Delta\omega_1 + \Delta\omega_2))T_b/2)}{(\omega - (\Delta\omega_1 + \Delta\omega_2))T_b/2}\right]^2 \qquad \text{... ... (9)}$$

Similarly,

$$P_2(f) = A^2 T_b\left[\frac{\sin((\omega - (\Delta\omega_1 - \Delta\omega_2))T_b/2)}{(\omega - (\Delta\omega_1 - \Delta\omega_2))T_b/2}\right]^2 \qquad \text{... ... (10)}$$

$$P_3(f) = A^2 T_b\left[\frac{\sin((\omega + (\Delta\omega_1 + \Delta\omega_2))T_b/2)}{(\omega + (\Delta\omega_1 + \Delta\omega_2))T_b/2}\right]^2 \qquad \text{... ... (11)}$$

and

$$P_4(f) = A^2 T_b\left[\frac{\sin((\omega + (\Delta\omega_1 - \Delta\omega_2))T_b/2)}{(\omega + (\Delta\omega_1 - \Delta\omega_2))T_b/2}\right]^2 \qquad \text{... ... (12)}$$

Adding equations (9), (10), (11) and (12) we get the power spectral density of complex binary signal.

$$P(f) = A^2 T_b\left[\text{sinc}^2\pi(f - \Delta f')T_b + \text{sinc}^2\pi(f - f'')T_b + \text{sinc}^2\pi(f + \Delta f'')T_b + \text{sinc}^2\pi(f + \Delta f'')T_b\right] \qquad \text{... ... (13)}$$

where, $\text{sinc}\, x = \dfrac{\sin x}{x}$

and

$$\Delta f' = \Delta f_1 + \Delta f_2$$
$$\Delta f'' = \Delta f_1 - \Delta f_2$$

Fig. 1. shows the variation of power spectral density with normalized frequency of FSK modulation.

Conclusion

The complex binary representation of signals are useful in generating complex low pass and band pass signals in straight forward manner. The variation of power spectral density with normalized frequency is shown in fig. (1). The results have been compared by plotting FSK (Proakis, 1983), GMSK (Murota and Hirade, 1981) and SFSK (Amoroso, 1976) conventional binary system. The power spectral curves are found close to the power spectra of GMSK modulation scheme which is applicable to global system for mobile communication and digital cellular services due to its bandwidth efficiency and constant envelope.

Table -1: Different values of a complex binary digit x_k

x_{rk}	x_{ik}	x_k
0	0	0
0	1	j
1	0	1
1	1	1+j

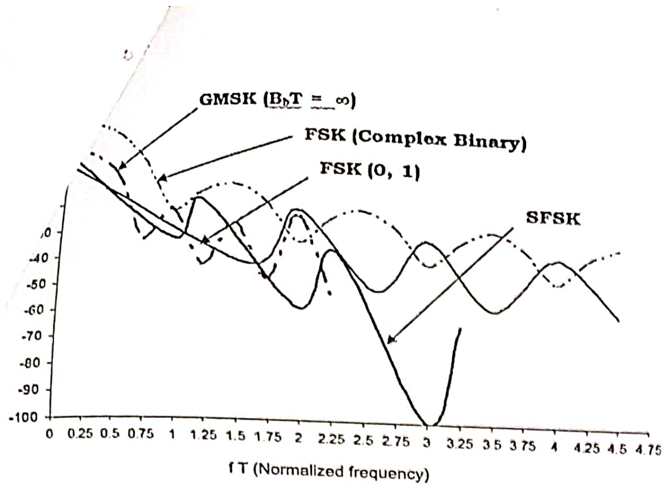

Fig. 1. Spectrum for Power Spectral Density of Complex Binary FSK Modulation

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