$$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 6\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{2}$$

(2) Harmonic Mean:

$$\frac{1}{H} = \int_0^1 \frac{1}{x}\, 6(x - x^2)\, dx$$

$$\text{or } \frac{1}{H} = 6\left[x - \frac{x^2}{2}\right]_0^1 = 3$$

$$H = 3$$

(3) Median:– M_d is given by

$$\int_0^{M_d} f(x)\,dx = \frac{1}{2}$$

$$\text{or } \int_0^{M_d} 6(x - x^2)\,dx = \frac{1}{2}$$

$$\text{or } 6\left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^{M_d} = \frac{1}{2}$$

$$4M_d^3 - 6M_d^2 + 1 = 0$$

$$(2M_d - 1)(2M_d^2 - 2M_d - 1) = 0$$

$$M_d = \frac{1}{2} \text{ or } M_d = (1 \pm \sqrt{3})$$

Since M_d lies between 0 and 1

$$\therefore M_d = \frac{1}{2}$$

$$\int_{-\infty}^{\infty} (x-m)^2 \, f(x)\, dx \cdot \text{ if } -\infty < x < \infty$$

$$= \int_a^b (x-m)^2 f(x) dx, \text{ if } a < x < b$$

Standard deviation S.D :- The positive square root of variance is called S.D and is denoted by σ.

Question :- For the distribution $dF = 6(x-x^2)\,dx$ $0 \le x \le 1$ find arithmetic mean, harmonic mean median, Mod, ~~mean deviation~~ and standard deviation.

Solution: For the given distribution we ha

$$\int_0^1 6(x-x^2)\,dx = \left[6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\right]_0^1 = 1$$

$\therefore$ $f(x) = 6(x-x^2)\,dx$ is a p.d.f

(1) Arithmetic Mean: $M = \int_0^1 x\, f(x)\, dx$

$$= \int_0^1 x \cdot 6(x-x^2)\,dx$$

$$= 6\int_0^1 (x^2 - x^3)\,dx$$

$$= 6 \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

Mathematical Expection or the Expected value:

Let x be discrete random variable and let its frequency distribution be as follows:

variate :	x_1	x_2	x_3	---	x_n
Probability :	P_1	P_2	P_3	---	P_n

where $P_1 + P_2 + P_3 + --- + P_n = \Sigma P = 1$

Then the mathematical expection of x (or simply expection of x) is denoted by $E(x)$ and is defined by

$$E(x) = P_1 x_1 + P_2 x_2 + P_3 x_3 + --- + P_n x_n$$

$$= \sum_{i=1}^{n} P_i x_i = \Sigma P x.$$

If $f(x)$ is a probability density function corresponding to the variate x Then

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

If $\phi(x)$ is a such function of x that it takes values $\phi(x_1), \phi(x_2)$ --- when x takes the values of x_1, x_2 ---

Let P_1, P_2, --- be their probabilities then the mathematical expection of $\phi(x)$, denoted by $E[\phi(x)]$ is defined as

$$E[\phi(x)] = P_1 \phi(x_1) + P_2 \phi(x_2) + --- + P_n \phi(x_n)$$

Product of Expections

Theorem: The expection of the product of two independent variate is equal to the product of their expections.

ie $$E(xy) = E(x) \cdot E(y)$$

Proof: Let x and y be two independent random variables. Let x assume m values x_1, x_2 --- x_m with corresponding probabilities p_1, p_2 --- p_m. Let y assume n values y_1, y_2 --- y_n with corresponding probabilities p_1', p_2' --- p_n'. Thus p_i is the probability of x assuming the value x_i and p_J' is the probability of y assuming the value y_J. Since x and y are independent, the probability that x assume the value x_i and at the same time y assumes the value y_J is $= p_i p_J'$ (By theorem of compound probability)

$\therefore$ By definition we have

$$E(xy) \sum_{i=1}^{m} \sum_{J=1}^{n} p_i p_J' x_i y_J$$

$$= \sum_{i=1}^{m} p_i x_i \sum_{J=1}^{n} p_J' y_J$$

$$= \sum_{i=1}^{m} p_i x_i \, E y$$

Result: (I) The covariance of two independent variates is zero.

(II) If the variance of variates $x_1, x_2, \cdots x_n$ are $\sigma_1^2, \sigma_2^2 \cdots \sigma_n^2$ respectively then the variance of u -

$$var(u) = a_1^2 \, var(x_1) + a_2^2 \, var(x_2) + \cdots + a_n^2 \, var(x_n) + 2a_1 a_2 \, cov(x_1, x_2) + \cdots + 2a_{n-1} a_n \, cov(x_{n-1}, x_n)$$

where $a_1, a_2, \cdots a_n$ are constants.

$$= 6 \times \frac{1}{120} = \frac{1}{20} = 0.05$$

(6) <u>standard Deviation</u>:

Variance $\sigma^2 = \int_0^1 (x-M)^2 f(x)\, dx$

$$= \int_0^1 \left(x-\frac{1}{2}\right)^2 6(x-x^2)\, dx$$

$$= 6\int_0^1 \left(x^2-x+\frac{1}{4}\right)(x-x^2)\, dx$$

$$= 6\int_0^1 \left(-x^4+3x^3-\frac{5}{4}x^2+\frac{1}{4}x\right) dx$$

$$= 6\left[-\frac{1}{5}x^5+\frac{9}{4}x^4-\frac{5}{12}x^3+\frac{1}{8}x^2\right]_0^1$$

$$= 6 \times \frac{1}{120} = \frac{1}{20} = 0.05$$

$\therefore$ standard deviation $\sigma = \sqrt{(0.05)}$

$= 0.223$

Ans

$$= E(y) \sum_{i=1}^{m} p_i x_i = E(y) \, (E(x))$$

$$\boxed{\therefore E(xy) = E(x)\, E(y)}$$

Covariance Definition

Let x and y be two random variables and $\bar{x}$ and $\bar{y}$ be their expected value (or mean) respectively. The covariance between x and y denoted by co(x, y) is defined as

$$co(x, y) = E\left[(x - \bar{x})(y - \bar{y})\right]$$

Thus the covariance between x and y is the product of their deviation from the mean.

(4) Mode :- It is that value of variate x for which

$$\frac{dy}{dx} = 0 \quad \text{and} \quad \frac{d^2y}{dx^2} < 0$$

Here $y = 6(x - x^2)$

$$\therefore \frac{dy}{dx} = 6(1 - 2x)$$

and $$\frac{d^2y}{dx^2} = 6(-2) = -12$$

$$\therefore \frac{dy}{dx} = 0 \Leftrightarrow 6(1 - 2x) = 0$$

$$\Rightarrow x = \frac{1}{2}$$

So mode To given distribution is $x = \frac{1}{2}$

Since mean, median and Mode coincide therefore the distribution is symmetrical.

(5) Mean deviation about the mean

$$= \int_0^1 \left|x - \frac{1}{2}\right|^2 6(x - x^2)\, dx$$

$$= 6 \int_0^1 \left(x^2 - x + \frac{1}{4}\right)(x - x^2)\, dx$$

$$= 6 \int_0^1 \left(-x^4 + 2x^3 - \frac{5}{4}x^2 + \frac{1}{4}x\right) dx$$

$$= 6 \left[-\frac{1}{5}x^5 + \frac{2}{4}x^4 - \frac{5}{12}x^3 + \frac{1}{8}x^2\right]_0^1$$

where $\Sigma P = 1$

Mathematical Expection for continuous Random variable.

Suppose x is a continuous random variable with probability density function $f(x)$, then mathematical expection $E(x)$ of x with certain restrictions is given by

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Expection of a sum

Theorem: The expection of the sum of two variate is equal to the sum of their expections ie if x and y are two variates then

$$E(x+y) = E(x) + E(y)$$

Proof: Let $x_1, x_2, -- x_m$ be the values of the variate P and $P_1, P_2 -- P_m$ be the corresponding probabilities. Also let $y_1\, y_2 -- y_n$ be the n values of y and $P_1', P_2' --- P_n'$ be the corresponding probabilities then the sum $x+y$ is a random variable which can assume mn values. $x_i + y_j$ ($i = 1, 2 --- m$; $j = 1, 2 -$

Let P_{iJ} be the probability of the variate x when x assumes the value x_i and variate y assumes the value y_J.

Since x takes a definite value x_i, y takes one of the n values $y_1, y_2 --- y_n$ so that the sum $\sum_{i=1}^{n} P_{iJ}$ represents the probability P_i of x assuming the values x_i, that is, we shall have

$$\sum_{J=1}^{n} P_{iJ} = P_i \quad \text{———} \quad (1)$$

Similarly discussing as above $\sum_{i=1}^{m} P_{iJ}$ represents the probability of y assuming the value y_J, that is, we shall have

$$\sum_{i=1}^{m} P_{iJ} = P'_J \quad \text{———} \quad (2)$$

Thus by definition we have

$$E(x+y) = \sum_{i=1}^{m} \sum_{J=1}^{n} P_{iJ}(x_i + y_J)$$

$$= \sum_{i=1}^{m} \sum_{J=1}^{n} P_{iJ}\, x_i + \sum_{i=1}^{m} \sum_{J=1}^{n} P_{iJ}\, y_J$$

$$= \sum_{i=1}^{m} \left(x_i \sum_{J=1}^{n} P_{iJ} \right) + \sum_{J=1}^{n} y_J \left(\sum_{i=1}^{m} P_{iJ} \right)$$

$$= \sum_{i=1}^{m} x_i P_i + \sum_{J=1}^{n} y_J P'_J \quad (\text{from } (1) \,\&\, (2))$$

Hence $E(x+y) = E(x) + E(y)$ ——— (3)

Product of Expections

Let x and y be two random variables. If x assume values $x_1, x_2, --- x_m$ with corresponding probabilities $f(x_1), f(x_2) --- f(x_m)$ and y assume values $y_1, y_2 -- y_n$ with corresponding probabilities $g(x_1), g(x_2) --- g(x_n)$ then the system of relations

$$P(x = x_i, y = y_j) = h(x_i, y_J)$$

is defined as the joint (or compound) probability distribution.

Independent variates:

Two random variates x and y on a sample space are called independent if the probability that either of them will assume a prescribed value does not depend on the value assumed by the other.