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(A Central University)

Department of Mathematics and Statistics, (July-Dec 2019)

Programme Name B.Sc. II Sem Section B
Semester: II
Course Code MTS CC- 211   Differential Equation   Course Credit   6
Course Title Differential Equation

S. N.	Semester Registration	Name of the Student		Assignment
1	Y19170003	Abhay Pandey		Continuous of function of one variable
2	Y19170007	Abhishek Shukla		
3	Y19170010	Aditya Patel		
4	Y19170011	Aditya Shivhare		
5	Y19170014	Adnan Khan Rangrej		Differentibiblty of function of one variable
6	Y19170018	Alka ladiya		
7	Y19170030	Ansul Kurmi		
8	Y19170032	Anuj Prajapati		
9	Y19170040	Aryan Trivedi		Exact Equations
10	Y19170047	Avinash Moorya		
11	Y19170048	Ayushi Jain		
12	Y19170049	Ayushi Mehagiya		
13	Y19170051	Basu Dubey		Rules of Integrating factors
14	Y19170054	Brajesh Beragi		
15	Y19170059	Deepali Bhargav		
16	Y19170068	Gourav Rai		
17	Y19170071	Hariram Ahirwar		Diggerential equation of solution for y
18	Y19170077	Hritik Dubey		
19	Y19170080	Jay Narayan Patel		
20	Y19170082	Jooli Prajapati		
21	Y19170085	Kamlesh Patel		Diggerential equation of solution for x
22	Y19170086	Kanika Shrivastava		
23	Y19170090	Krishnakant Kandar		
24	Y19170099	Manish Mishra		
25	Y19170101	Mansi Rathore		Diggerential equation of solution for p
26	Y19170103	Meena Raj Gound		
27	Y19170106	Mitika Jain		
28	Y19170108	Mohandas Prajapati		
29	Y19170112	Namrata Sahu		singular solutions
30	Y19170115	Neha Bai Gound		
31	Y19170121	Palak Litoriya		
32	Y19170123	Pooja Singh Gound		
33	Y19170125	Pranjalee Singh Dixit		Diggerential equation of forst degree and first order
34	Y19170128	Pratima Singh		
35	Y19170132	Priya Pachouri		

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Programme Name B.Sc. II Sem Section B
Semester: I
Course Code MTS CC- 211 Differential Equations Course Credit 6

S. N.	Semester Registration No.	Name of the Student		Assignment
36	Y19170133	Priyansh Agrawal		linear homogeneous equation with constant coefficient
37	Y19170135	Puhpendra umar Lodh		
38	Y19170140	Rahul Shrivastava		
39	Y19170141	Rahul K Vishwakarma		
40	Y19170143	Ram Singh Rajpoot		linear homogenious equation
41	Y19170158	Sajal Kurmi		
42	Y19170159	Saket Thakur		
43	Y19170162	Sameer Sen		
44	Y19170163	Sanskriti Jain		simultateous equations
45	Y19170165	Santosh Singh Lodhi		
46	Y19170166	Santram Sahu		
47	Y19170171	Satya Prakash Verma		
48	Y19170173	Sejal Soni		partial differential equation
49	Y19170175	Sevendra Ahirwar		
50	Y19170187	Shubhum Mishra		
51	Y19170189	Shubhi Saini		
52	Y19170190	Siddhant Singh Rajpoo		theorems on diferntebility of functions of one variable
53	Y19170192	Somya Gupta		
54	Y19170198	Spardha Wadekar		
55	Y19170200	Sumit Namdeo		
56	Y19170201	Suraj Singh Thakur		proof of complimentry solutions of linear equations
57	Y19170202	Surbhi Sahu		
58	Y19170204	SusheelAhirwar		
59	Y19170205	Susheel Vishwakarma		
60	Y19170207	Tanish Chourasiya		
61	Y19170218	Yogesh Yadav		

Wronskian.

Definition:- suppose a linear differential equation is $\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = 0$ has two solution $y_1(x)$ and $y_2(x)$, then their Woronskian denoted by $W(x)$ and define as

$$W(x) = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}$$

One can extend the definition for nth order.

If $y_1(x), y_2(x), \dots, y_n(x)$ are n solutions of LDE $y^{(n)} + P_1(x)y^{(n-1)}(x) + \dots + P_{n-1}(x)y'(x) + P_n(x)y = 0$

then Wronskian defined as

$$W(x) = \begin{vmatrix} y_1(x), & y_2(x) & \dots & y_n(x) \\ y_1'(x) & y_2'(x) & \dots & y_n'(x) \\ \vdots & & & \\ y_1^{(n-1)}(x) & y_2^{(n-1)}(x) & \dots & y_n^{(n-1)}(x) \end{vmatrix}$$

orem -

Linearly Independent - If $y_1(x), y_2(x), \dots y_n(x)$ are n solutions of a differential equation, then they are called Linearly Independent if

$$c_1y_1(x) + c_2y_2(x) + \dots + c_ny_n(x) = 0$$

them $c_1 = c_2 = \dots = c_n = 0$

Linearly Dependent - If $y_1(x), y_2(x) \dots y_n(x)$ are n solutions of a LDE, them they are called L.D. if $c_1 y_1(x) + \dots + c_n y_n(x) = 0$

Then at least one $a_i \neq 0$, $i = 1, 2 \dots n$.

Theorem: If Wronskian of a D.E. is non zero at a point, then solutions are Linearly Independent.

Proof:- suppose a D.E. $y^{(n)}(x) + P_1(x) y^{(n-1)}(x) + \dots + P_{n-1}(x) y'(x) + P_n(x) y = 0$ has n solutions $y_1(x), y_2(x), \dots$ & $y_n(x)$.

If them

$$c_1y_1(x) + c_2y_2(x) + \dots + c_ny_n(x) = 0$$

$$c_1y_1'(x) + c_2y_2'(x) + \dots + c_ny_n'(x) = 0$$

$$c_1y_1''(x) + c_2y_2''(x) + \dots + c_ny_n''(x) = 0$$

$$\vdots$$

$$c_1y_1^{(n-1)}(x) + c_2y_2^{(n-1)}(x) + \dots + c_ny_n^{(n-1)}(x) = 0.$$

To write in matrix form we have

$$\begin{vmatrix} y_1(x) & y_2(x) & \dots & y_n(x) \\ y_1'(x) & y_2'(x) & \dots & y_n'(x) \\ \vdots & & & \\ y_1^{(n-1)}(x) & y_2^{(n-1)}(x) & \dots & y_n^{(n-1)}(x) \end{vmatrix} \begin{vmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{vmatrix} = 0$$

Given that $w(x) \neq 0$; or $w(x)\, C = 0$ —①

where $w(x) = \begin{vmatrix} y_1(x) & \dots & y_n(x) \\ y_1'(x) & \dots & y_n'(x) \\ y_1^{(n-1)}(x) & \dots & y_n^{(n-1)}(x) \end{vmatrix}$ and $C = \begin{vmatrix} c_1 \\ \vdots \\ c_n \end{vmatrix}$.

Sin $w(x) \neq 0$, then $C = 0$

i.e $c_1 = 0, c_2 \dots c_n = 0$

hence $y_1(x), y_2(x), \dots y_n(x)$ are L.I.

Unit 3 का शेष

Calculus of variations

It mean a quantity whose values are determined by one or more fuctions and theory of maxima and minima of fuctions. of one or more variables.

If $y = y(x)$ represents a path joining two fixed points $A(x_1, y_1)$ and $B(x_2, y_2)$ and $F(x, y, y')$ is a function the

$I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\,dx$ is called Euler's Equation (I)

Condition of Euler Equation is $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$ (II)

Q1. investigate for an extremum the functional

$$I[y(x)] = \int_0^1 \left(2 + 2y + \frac{y'^2}{2}\right) dx, \quad y(0) = 0, \ y(1) = 0$$

Solution On compare this with equation ①

we get $F(x, y, y') = 2 + 2y + \frac{y'^2}{2}$

According to Euler's Equation (II)

$\frac{\partial F}{\partial y'} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$

$\Rightarrow 2 - \frac{d}{dx}(y') = 0$

$2 - y'' = 0$

$y'' = 2$

on integrating w.r.t x

$y' = 2x + C_1$

Again integrating w.r.t x

$\boxed{y = x^2 + C_1 x + C_2}$

we have to find C_1 & C_2 with the help of Boundary condition

which are $y(0) = 0$

on putting in (III)

we get $0 = C_2$ — (IV)

~~on pu~~ and $y(1) = 0$

on putting in (III)

$0 = 1 + C_1 + C_2$ — (V)

from (IV) & (V) Equation

we get $C_2 = 0$ & $C_1 = -1$

hence the solution will be

$y = x^2 + C_1 x + C_2$

or $\boxed{y = x^2 - x + 0}$

Home work. Test for an extremum the functional

1) $I[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)\, dx, \quad y(0) = 0, \quad y(\pi/2) = 1$

2) $I[y(x)] = \int_1^2 \frac{x^3}{y'^2}\, dx, \quad y(1) = 1, \quad y(2) = 4.$

3) $I[y(x)] = \int_0^2 (e^{y'} + 3)\, dx; \quad y(0) = 0, \quad y(2) = 1$

4) $I[y(x)] = \int_0^1 (xy + y^2 - 2y^2 y')\, dx, \quad y(0) = 1, \quad y(1)$

Ans. 1) $y = \sin x$

2) $y = x^2$

3) $y = x/2$

4) $(2y - x)^2 (4y + x) = 16$
